

Modeling collective rule at ancient Teotihuacan as a complex adaptive system: Communal ritual makes social hierarchy more effective

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Abstract

Experts remain divided about the nature of the sociopolitical system of ancient Teotihuacan, which was one of the earliest and largest urban civilizations of the Americas. Excavations hoping to find compelling evidence of powerful rulers, such as a royal tomb, keep coming away empty-handed. But the alternative possibility of collective rule still remains poorly understood as well. Previously we used a computational model of this city's hypothetical sociopolitical network to show that in principle collective rule based on communal ritual could be an effective strategy of ensuring widespread social coordination, as long as we assume that the network's structure could be transformed via social learning and local leaders were not strongly subdivided. Here we extended this model to investigate whether increased social hierarchy could mitigate the negative effects of such strong divisions. We found a special synergy between social hierarchy and communal ritual: only their combination improved the extent of social coordination, whereas the introduction of centralization and top-down influence by themselves had no effect. This finding is consistent with portrayals of the Teotihuacan elite as religious specialists serving the public good, in particular by synchronizing the city's ritual calendar with the rhythms of the stars.

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1. Introduction

Teotihuacan was one of the earliest and largest civilizations in the ancient Americas, with an impressive urban center based in Central Mexico (Fig. 1). This multiethnic city was situated at the heart of a far-reaching network of ideological, economic, and political influence until its collapse in the 6–7th century CE (for a recent general intro-

duction, see Robb (2017); for comprehensive scholarly accounts, see Cowgill (2015), Nichols (2016), and Manzanilla (2017b)). The site has long been a focus of interest in Mesoamerican archaeology, and its scale and early emergence mean that it is also a key site for comparative studies of urbanism and state formation (Carballo, 2016; Kohler et al., 2017; Manzanilla, 2007, 2017b; Ortman, Cabaniss, Sturm, & Bettencourt, 2014). A variety of archaeological projects is slowly filling in more details about the city, both at the level of its many neighborhoods (Gómez-Chávez (2012); Manzanilla, 2009a, 2012, 2017a;

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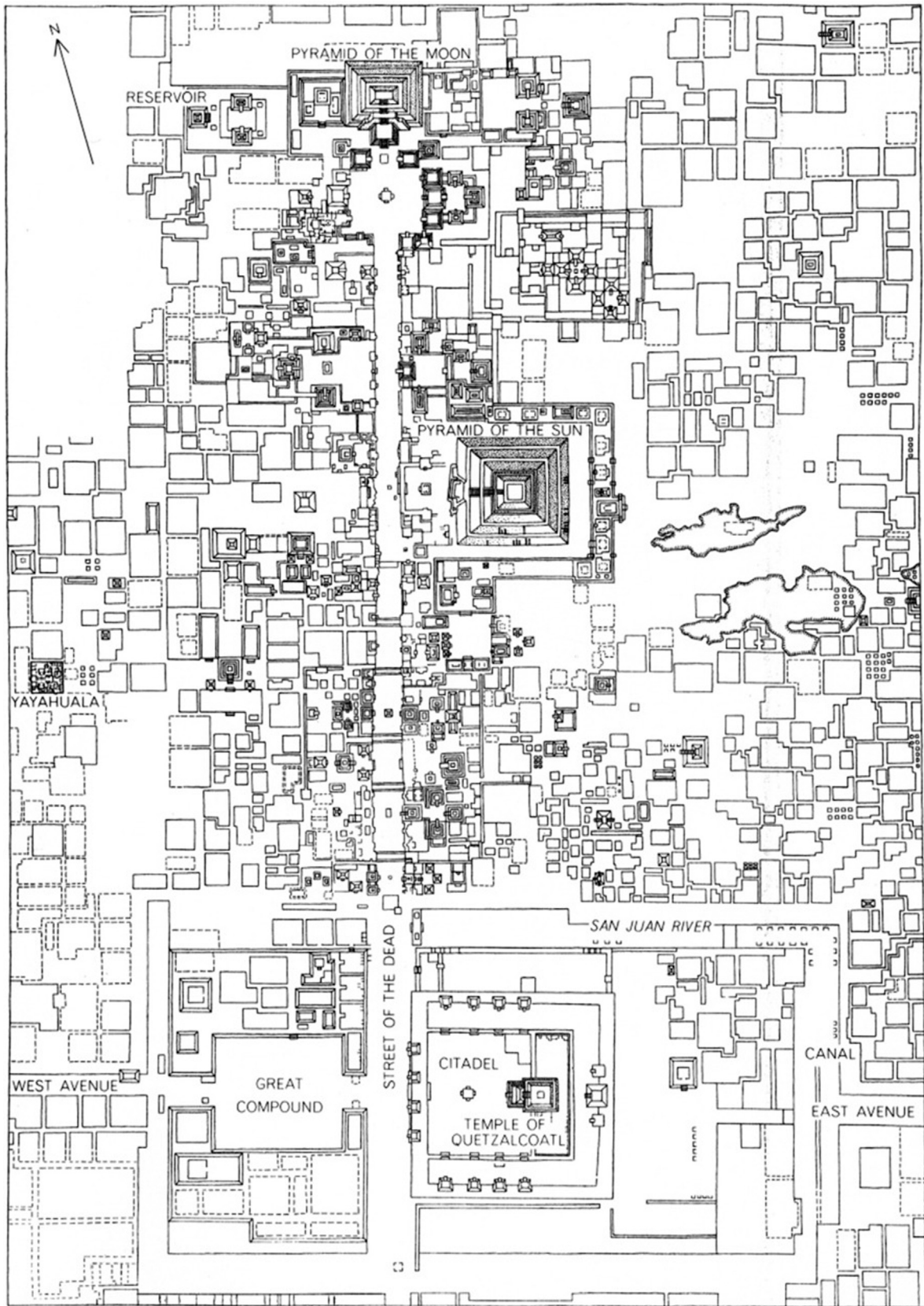


Fig. 1. Map of the city center of ancient Teotihuacan. This map was produced by the Teotihuacan Mapping Project coordinated by Millon (1973). Some of the main features of the center that are discussed in the text are labeled.

Widmer & Storey, 2012) and of its ceremonial core (Sugiyama, Sugiyama, & Sarabia, 2013; Sugiyama, 2005).

However, surprisingly, the most immediately visible aspects of many other ancient urban civilizations – its rulers and their palaces – have proven difficult to identify and there is still little consensus about them (Evans, 2006; Manzanilla, 2001, 2008; Nielsen, 2014; Sanders & Evans, 2006; Smith, 2017). Hypotheses about Teotihuacan's main mode of government can be grouped into two categories: (1) the city had individualized rule based on a dynastic lineage of powerful kings (Coe & Koontz, 2013; Headrick, 2007; Millon, 1993; Sugiyama, 2005), and (2) the city had a collective government based on extensive power sharing among various groups and different levels of society (Angulo, 2007; Blanton, Feinman, Kowalewski, & Peregrine, 1996; Manzanilla, 1992, Manzanilla, 2008, 2015, 2017b; Nichols, 2016; Pasztory, 1997; Paulinyi, 1981, 2001).

The disagreement between scholars is most prominent regarding the mode of government of the earliest phases of the city, about which very little is known. There is evidence that different groups came together to found the city, which would be suggestive of collective rule, but this initial phase culminated in the construction of the highly organized monumental architecture in its ceremonial core, which for many scholars is indicative of powerful individual leaders. This initial phase of the city will be the focus of this article and we aim to challenge the common intuition that the emergence of such complex social order requires individualized rule in order to enforce citywide social coordination. Instead, we agree with Stanish's (2017) theoretical framework, according to which ritual practice is the primary mechanism of organizing society in the absence of a coercive state apparatus.

Our effort can be seen as contributing to a growing movement within Mesoamerican archaeology that is contesting the traditional default assumption, namely that ancient government consisted in individualized rule of powerful autocrats who tried to impose pervasive top-down control. That restrictive focus on hierarchy is being replaced with a broader perspective on heterarchy, which emphasizes the role of economic interdependency and political cooperation in the development of ancient statecraft (Crumley, 2003), and draws on insights from the evolution of cooperation and collective action literature (Carballo & Feinman, 2016; Carballo, Roscoe, & Feinman, 2014; Fargher, Heredia Espinoza, & Blanton, 2011).

Froese has argued that this shift in thinking is also supported by recent advances in complex systems theory, and can be further aided by the use of computer simulations (Mezza-Garcia, Froese, & Fernández, 2014; Ulloa & Froese, 2016). These advances demonstrate how the creation of social order can be “out-sourced” from individuals' internal cognitive processes into their extended social interactions, thereby reducing the necessity of relying in powerful institutions. Even ritual practices involving intox-

ication, which can be so extreme that some scholars consider them maladaptive, can be fruitfully reinterpreted in terms of complex adaptive systems that make use of disorder to produce order (Froese, 2015). The hypothesis of collective rule at Teotihuacan can be usefully approached from this interdisciplinary perspective, which highlights the key role played by such communal rituals in giving rise to learning at the level of social networks (Froese, *in press*).

Given that little is known about Teotihuacan's form of government, especially during the early phases, there is an opportunity to employ computational techniques to explore this space of possibilities at a relatively abstract level of description. Previously we created a model to investigate the role of communal ritual as a mechanism of social integration at the level of neighborhoods in the absence of individualized rule (Froese, Gershenson, & Manzanilla, 2018). We simulated a possible social network of neighborhood rulers, and tested the efficacy of communal ritual to integrate them into a larger, coordinated whole.

In particular, we focused on extreme rituals: there are mural paintings of a kind of ritual activity that, in contrast to representations of other ritual activity such as processions, do not include any identifiable individuals or social roles that would permit one to infer hierarchical relations. To the contrary, they show a large number of anonymous participants engaged in a variety of often relatively unconstrained activities, sometimes with a peculiar emphasis on intoxication to the extent of vomiting and loss of motor control (Angulo, 1995; Cabrera Castro et al. (2007); Nielsen & Helmke, 2017; Paulinyi, 2014). The simulation model illustrated how such periods of ritualized “anti-structure” (Turner, 1969) could have enabled the social network to implicitly learn about its space of possible configurations, allowing it to generalize toward configurations that facilitate large-scale social coordination. The model also revealed that this effect would have been impeded by social divisions created by the clustering of the neighborhood centers into the quadrants of the city,¹ which may have been one factor causing the city's eventual collapse.

In this article we build on these results and explore whether increasing centralization and/or increasing levels of social hierarchy could help to counteract the impediment to ritual-based social learning that is posed by increasing neighborhood clustering.

2. Previous work

It has been proposed by various scholars that an early form of Teotihuacan's collective government may have been realized at the level of neighborhood temple centers, especially those known as Three-Temple Complexes, and that it was integrated by ritual (Angulo, 2007; Manzanilla, 1997; Pasztory, 1988). The triadic format of

¹ We will use the terms district and quadrant interchangeably throughout this article.

these neighborhood centers has antecedents in other areas of Mesoamerica before the city's foundation (Plunket & Uruñuela, 1998), and it has been linked with a Mesoamerican creation myth centered on the setting of three stones (Headrick, 2007, pp. 103–118). The precise form and role of the rituals in the realization of collective rule remains unclear, but it is widely agreed that Teotihuacan's influential religious ideology (Filini, 2015) and its pervasive ritual activity, such as processions (Evans, 2016), helped to integrate the city's population. A couple of Three-Temple Complexes are shown in Fig. 2.

We identified 22 Three-Temple Complexes of comparable size based on Millon's (1970) map of the city, which is

consistent with other counts reported in the literature. This number is also reflected in Millon's (1988, p. 91) interpretation of mural paintings of a procession as possibly involving the heads of 20 kin groups participating in the city's founding. Intriguingly, this number is also consistent with the fact that during the 16th century, long after the collapse of the ancient city, the Spanish recorded approximately 20 tributary settlements surrounding the surviving town of Teotihuacan, where that region's Aztec ruler lived (Hirth, 2008). Moreover, Hirth's analysis provides a potential response to the worry that the uneven, centrally clustered distribution of the Three-Temple Complexes makes it unlikely that they were neighborhood centers (e.g. Cowgill,

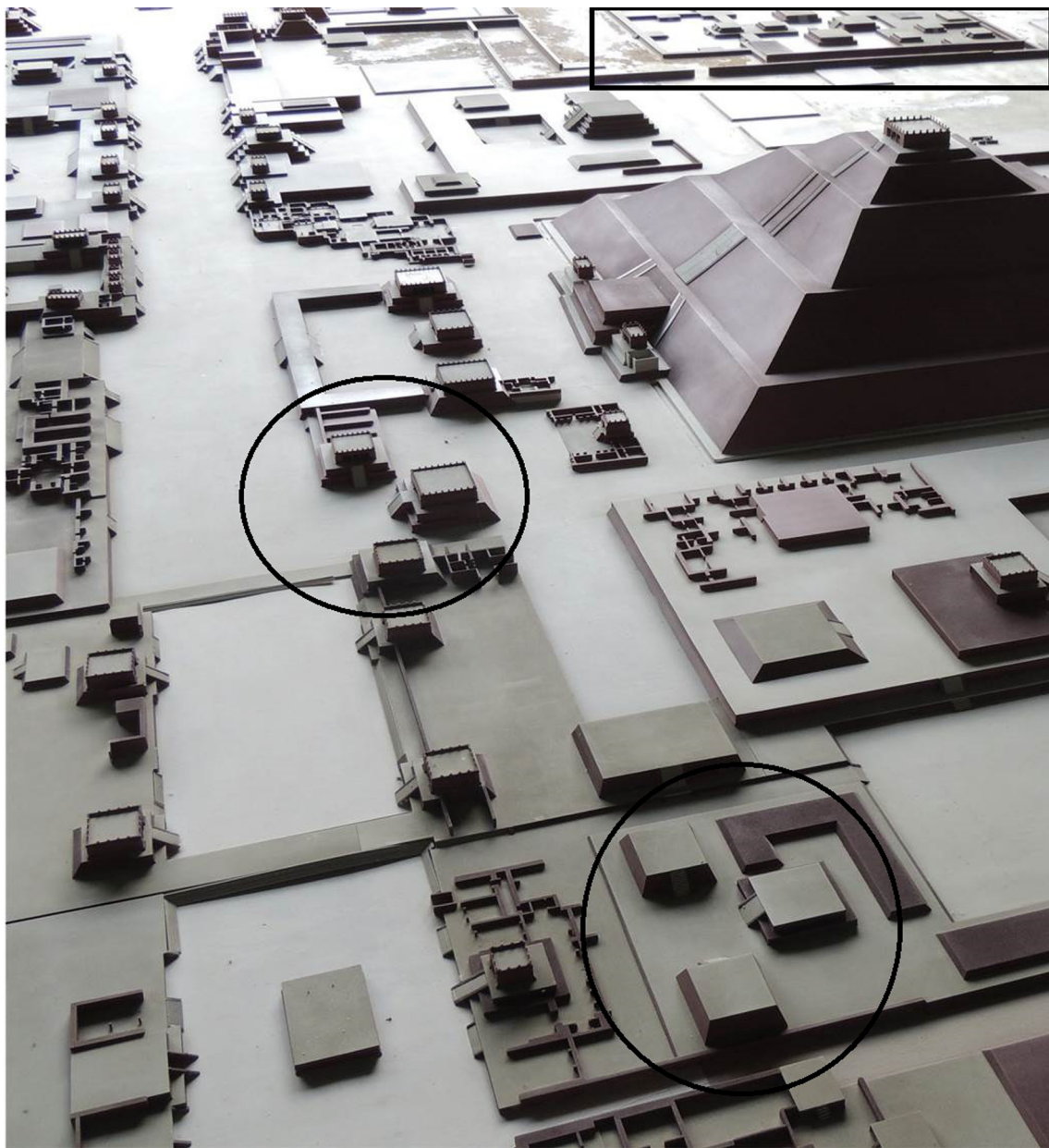


Fig. 2. Illustration of typical Three-Temple Complexes. Central part of a mockup of Teotihuacan's ceremonial core located in the Museo de Sitio de la Cultura Teotihuacana, Teotihuacan. There are two Three-Temple Complexes (circled in black), while the Xalla compound, a possible seat of early government, is also visible in the distance above the Pyramid of the Sun (highlighted in a rectangle). Photo adapted from Froese et al. (2018).

2008): at least during the later Aztec period the administrative centers of the segments of a region sometimes formed a loosely integrated city-like cluster while actual land holdings were distributed and disarticulated.

In order to increase the complexity of the coordination problem faced by this collective rule scenario, we arbitrarily assumed that there were three persons in charge at each Three-Temple Complex, a decision that is loosely based on their triadic format. We assumed that there were symmetrical relations between leaders. However, the strength of this mutual influence was unequally distributed throughout the network: we assumed that the three leaders of a Three-Temple Complex were most tightly coupled, while constraints between leaders from different Complexes were weaker. We assumed that every leader is connected with every other leader (hence forming a fully connected network), but most connections are actually very weak. The resulting local clustering makes it hard for the agents to coordinate behaviors across the whole network.

We compared this initial “Neighborhoods” scenario, in which Three-Temple Complexes were still not clustered into the city’s quadrants, with a “Districts” scenario (see map in Manzanilla, 2009a), in which connections between leaders of Three-Temple Complexes in the same quadrant had become more important compared to connections with leaders from other quadrants. We assumed that there were four districts that were separated by a vertical axis running along the Avenue of the Dead and a horizontal axis that was originally across the Pyramid of the Sun, which meant that the neighborhood centers were divided as follows: Northwest (10), Northeast (2), Southwest (5), and Southeast (5).

For simplicity, each agent in the network can decide to adopt only one of two behaviors. We defined social coordination as the resolution of social conflicts, which in this model takes the form of one agent matching the behavior of another agent. We know from the ethnographic record that such consensus formation was an important component of social complexity in stateless societies (Stanish, 2017). Coordination is a symmetrical notion since both agents benefit from a resolution of their conflict. This differs from the notion of cooperation employed in game theory, which typically requires an asymmetrical relation between two or more agents because the other agents’ benefit from cooperation is associated with a cost or risk for the first agent (see, e.g., Axelrod, 1984; Carballo et al., 2014).² Nevertheless, symmetry is not sufficient to ensure coordination among all agents, since the optimal strategy of an individual agent often differs from the optimal strategy of its group. An agent always updates its behavior and connections with the aim of minimizing its current perceived conflicts, which in turn places constraints on the configurations that the group as a whole can explore: often

a configuration with fewer conflicts cannot be reached because it would require some of the agents to give up being selfish, that is, to behave so as to temporarily increase the extent of their conflicts. The model does not permit such altruistic behavior and reaching optimal configurations is therefore highly unlikely, at least without the help of additional processes.

The same applies to changes in social relations: agents can strengthen their connections to other agents with whose behavior they are in agreement, while weakening their connections to other agents with whose behavior they are in conflict. However, they will always adjust their relations in a selfish manner so as to maximize their own benefit, and so there is a network-wide coordination problem also at the level of structural changes.

The solution to this coordination problem are periodic communal rituals, which are implemented as a generalized ‘reset’ of the whole social system by temporarily setting the behavior of the agents to an arbitrary configuration (Froese et al., 2018; see also Watson, Mills, & Buckley, 2011). This reset allows the network to become unstuck from unfavorable behavioral configurations and converge on alternative configurations, some of which may be preferable. More importantly, this exploration of different configurations, combined with the agents’ structural reinforcement of those configurations via changes in their social relations, has the positive effect that the social system as a whole will start to learn and recall the better configurations it has visited in the past, and even begin to generalize over them in a way that facilitates encountering better solutions that had not yet been visited (Watson, Mills, et al., 2011). What is interesting is that this social learning is happening at the level of the network as a whole without any individual agent being in charge: no top-down control of the whole system is needed for this social learning, nor does it require any knowledge of what the optimal configuration actually consists in.

In summary, the dynamics of this model minimally capture the interaction between individual agency and supra-individual processes by including three distinct timescales (from fastest to slowest): (1) selfish updating of individual behavior, (2) selfish updating of social relations, and (3) collective resetting of all behaviors to arbitrary states via ritualized interventions. More specifically:

- (1) *Decision-making*: Each agent in the model will selfishly adopt one of two behavioral states, which represent any kind of binary choice (e.g. voting to build a large pyramid versus going on a raid). It is assumed that it is beneficial for all the neighborhood leaders if their behaviors align to form a consensus.
- (2) *Learning*: Each agent is able to selfishly adjust the relative strengths of their social relations so as to mitigate the impact of conflicts. This has the effect of reinforcing an agent’s likelihood of coordinating behaviors again with those agents with whom there were successful interactions in the past.

² It is an interesting open question how the mechanisms of social learning and communal ritual that we implemented in our coordination model could be translated into a game theoretic framework.

- (3) *Ritual*: There is a synchronized release of all agents' behavior from the structural constraints of normal interactions. Afterwards there is a period of re-convergence: behaviors update and slowly become aligned again with respect to the differing constraints posed by others' behavior. Following van Gennep (1908/1960) and Turner (1969), we can think of this intervention in terms of the three phases of a ritual: separation, liminality (anti-structure), and incorporation.

It is difficult to tie these model timescales to actual timescales expressed in months and years, but it seems reasonable to assume that there were at least two major ritual resets per year, for example to mark the transitions between rainy and dry seasons. Future work could try to improve the realism of the model, for example by increasing the agents' behavioral complexity and by adopting empirically grounded time scales. However, we emphasize that the principal aim of our contribution was not to create a realistic model of the Teotihuacan government, which would have been an impossible task given the lack of relevant data. Instead we aimed for an abstract model that is minimal enough to be analytically and computationally tractable, and yet complex enough to serve as a useful conceptual tool to advance the debate.

The model is initialized by setting the behaviors of all agents to one or the other decision state with equal probability, and this means that 49% of agents tend to be in agreement with each other by chance (we will call this the “Initial” configuration).³ These odds are consistently improved after agents are allowed to update their behaviors (the “Converged” configuration). However, as was expected, they always fail to converge on a network-wide consensus, revealing a problem of social coordination: the interest of the individual agents is not easily aligned with the interest of the collective group. This problem is especially notable in the “Districts” scenario, for which convergence of behaviors only improved conflict resolution on average by 10% over that found in a random distribution of behaviors (i.e. 59% of all connections are without conflict). Strong subdivisions make it even more difficult for the social network to converge on an optimal consensus because, as would be expected, leaders of one quadrant often converge on a consensus among themselves that is distinct from the consensus reached in the other quadrants.

However, importantly, in the “Neighborhoods” scenario the learnt changes in connections in combination

with ritual-based resets are able to successfully overcome these coordination problems (the “Optimized” configuration). This result is quite remarkable: leaders engaged in selfish decision-making and selfish learning nevertheless spontaneously end up coordinating their behavior in an optimal manner and reach a full consensus. In essence, this happens because the leaders occasionally jointly leave their normal constraints aside and behave in a ritualized, arbitrary manner, which allows the network as a whole to explore, learn about, and reinforce whatever new configurations of behaviors it eventually re-converges on. We emphasize again that there are no leaders directly in charge of this process. And even if there were leaders with sufficient power, they would not know how to facilitate the emergence of an optimal configuration because the combinatorial problem space is just too complex. In other words, to a population benefiting from such a ritually mediated process of spontaneous self-optimization it must have indeed looked like the leaders' rituals were efficacious and that the gods were favorably inclined towards them.

It is also noteworthy that the model offers a notion of social learning that goes beyond the observation that much individual learning is social because individuals acquire knowledge and know-how by participating in communities of practice (Wenger, 2000). On top of the structural changes enacted by individuals the model adds a complex adaptive systems perspective: a process of learning also takes place in a distributed manner at the level of the sociopolitical network as a whole, a process which is enabled by individual structural changes but irreducible to them, akin to how associative memory is realized at the level of a nervous system rather than just by individual neurons (Watson, Mills, et al., 2011).

There are limits, however. The positive effects of such ritualized self-optimization of the network's connectivity become impaired when the subdivisions in the network are too strong, as exemplified by the “Districts” scenario. Specifically, while the optimized configuration managed to resolve 99% of social conflicts in the “Neighborhoods” scenario, it only managed to resolve 80% in the “Districts” scenario. Accordingly, the “Districts” scenario provides a suitable starting point for an extension of the original model to investigate the capacity of increasing social hierarchy to recover higher levels of social coordination via top-down control. In particular, it is an opportunity to model Manzanilla's proposal that there may have been four co-rulers at the top of Teotihuacan's sociopolitical network. We next present an extension of the original model in this direction.

3. The model

This original network model was completely horizontal and revealed that increased clustering of the Three-Temple Complexes impeded social coordination despite ritual-based social learning, which leads to the plausible proposal that this impediment could be overcome by the introduc-

³ Chance cooperation is slightly less than 50% because for 66 agents the smallest possible number of cooperating agents is 33, i.e. 33 cooperate in doing 'a' and the remaining 33 cooperate in doing 'b'. Thus, the connections among the 33 'a' agents ($33 * 33 - 33 = 1056$) plus the connections among the 33 'b' agents ($33 * 33 - 33 = 1056$) will minimally be satisfied, giving a total of 2112 of connections, which is less than half of all connections ($(66 * 66 - 66)/2 = 4290/2 = 2145$).

tion of a higher level of social organization that reintegrates the clusters. The intuitive starting point is to incorporate a hypothetical palace into the model, even if there is little consensus on this topic among Teotihuacan scholars. One of the more likely contenders is the Xalla compound (Manzanilla, 2008, 2009b, 2017b; Manzanilla & López Luján, 2001), shown in Fig. 3, which was possibly a political center during the early phases of Teotihuacan (Evans, 2006; Manzanilla, 2001; Sanders & Evans, 2006). Manzanilla, who has been excavating this compound since the year 2000, has been suggesting that this large compound may have been the seat of four co-rulers of the four quadrants of the city, or perhaps only two of them as well as their two female counterparts (Manzanilla, 2017c), who held office at Xalla's central four-temple group.

Admittedly, these are just hypotheses at this point. Most of the Three-Temple Complexes remain unexcavated, and the excavations at Xalla by Manzanilla are still ongoing, so it remains to be more systematically determined in which period these structures were constructed and also what functions and interactions they had. In addition, the identification of neighborhoods, districts, and larger divisions in the city continues to be a challenging undertaking (Altschul, 1987; Manzanilla, 2009a, 2012; Robertson,

2015). Accordingly it is fair to say that considerable uncertainties remain about the extent to which the Three-Temple Complexes and/or Xalla played a role in the city's early government (see, e.g., Cowgill, 2015, p. 75, p. 116, pp. 119–122), and about whether the city was at some point governed by four co-rulers that were the most influential rulers of the city's quadrants (Nielsen, 2014; Villa Córdova, 2016). On the other hand, even if it were to turn out that Xalla was not the seat of a government of four co-rulers, the possibility of four founding rulers has also been considered based on recent excavations of the tunnel under the Ciudadela compound (Gómez Chávez, 2017, p. 54).

It is not our aim to enter further into this ongoing debate here. We simply assume for the sake of argument that the four quadrants of the city formed meaningful political districts (Manzanilla, 2017b). We also assume that these quadrants were associated with four co-rulers, who may have been based at the Xalla compound or perhaps elsewhere. We further assume that each ruler exclusively interacted with the leaders of the neighborhoods in their respective district, and that these local leaders were based at the Three-Temple Complexes. Our principal aim, like that of “artificial society” research more generally (Lansing, 2002), is to probe and challenge our intuitions

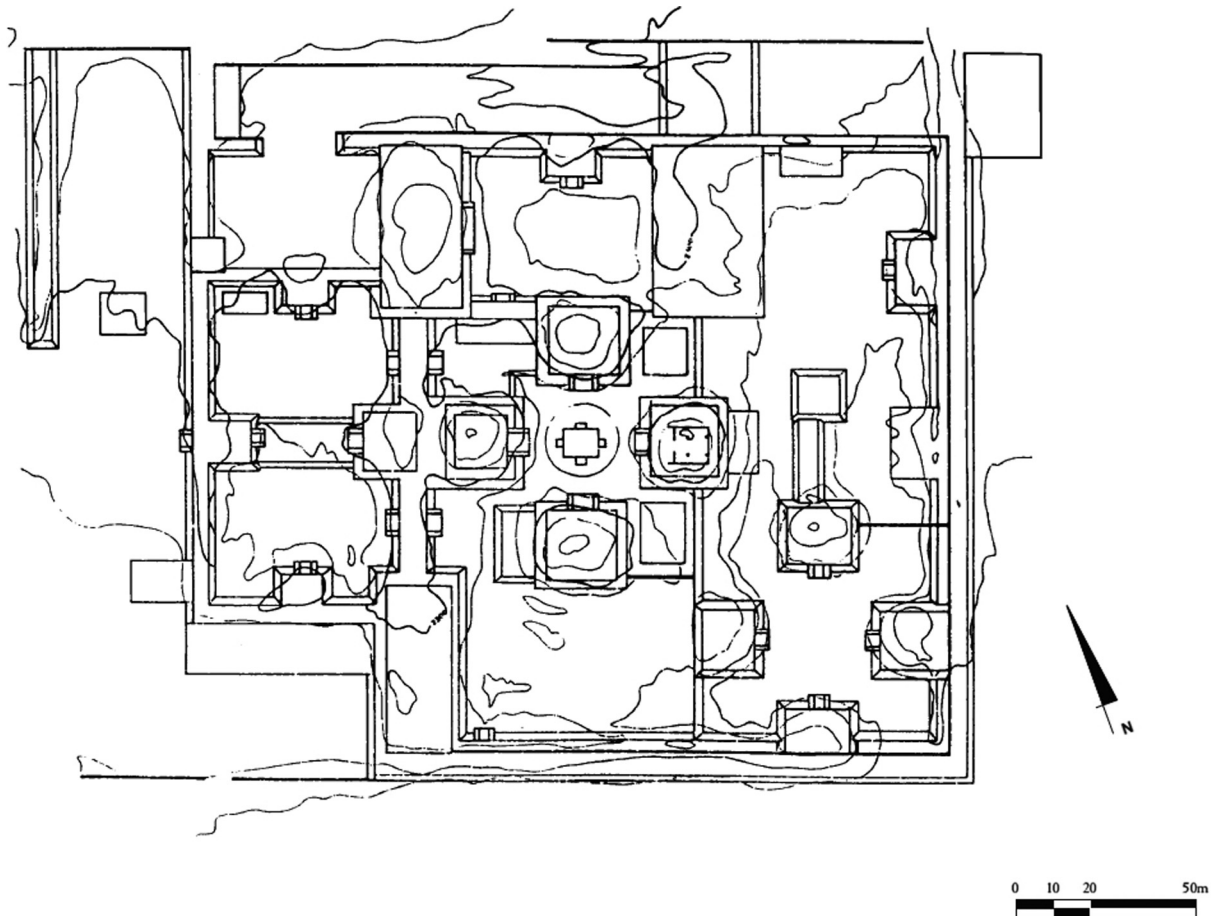


Fig. 3. Layout of the Xalla compound. This palace-like compound may have been one of the seats of Teotihuacan's collective government during the early phases of the integrated city. Figure taken from Manzanilla (2001).

about this scenario. The model will have achieved its purpose if it helps us to think differently about the city's origins of complex social order, and if it allows us to generate new hypotheses and questions that can guide empirical and art historical research at Teotihuacan.

We now describe the new model in plain English in order to make it more accessible to a broad audience, while the mathematical formalisms can be found in [Appendix A](#).

3.1. Network topology

We accommodated the hypothesis of four co-rulers proposed by Manzanilla by extending this original model as follows. We added four additional nodes, representing these higher-level co-rulers, to the existing network. Similar to the case of a Three-Temple Complex, primacy was assigned to satisfying the constraints imposed by the relations between the four co-rulers, given their assumed spatial co-presence at Xalla or elsewhere. However, in contrast to the original model, we allowed centralized and hierarchical relationships: only the neighborhood leaders of a co-ruler's quadrant are able to directly influence their co-ruler's behavior, and their co-ruler can similarly only directly influence the behavior of leaders from their own quadrant. We varied the level of the co-ruler's influence, ranging from equal to higher than that of a neighborhood leader's influence, in order to evaluate the effects of increasing centralization with and without hierarchical top-down control:

- The “*Co-rulers (weak)*” scenario investigates the effects of an increase in political centralization without a corresponding increase in hierarchy. Each co-ruler is connected with the neighborhood leaders of their district in the same way as those leaders are interconnected within their district. Connections are symmetrical.
- The “*Co-rulers (intermediate)*” scenario investigates a hierarchical and centralized network of co-rulers. The topology is almost the same as the one of the “*Co-rulers (weak)*” scenario except for the key difference that the co-rulers now exert five times more influence over the behavior of the leaders of their quadrant than vice versa. The network is therefore no longer fully interconnected and connections are allowed to be asymmetrical (i.e. the network is now a directed graph).
- The “*Co-rulers (strong)*” scenario increases co-rulers' top-down influence to a factor of ten: they exert ten times more influence over the behavior of the leaders of their quadrant than vice versa. The strength of this influence is largely an arbitrary choice, although it should be noted that, if we accept that compound size is an indication of wealth and power then this parameter makes sense because “Xalla covered 10 times as much space as would the average compound” ([Sanders & Evans, 2006, p. 261](#)).

The precise values of the network's parameters can be found in [Appendix A](#).

3.2. Model dynamics

Each scenario was tested in 200 independent simulation runs. Each run starts with a different random number seed. For each run, we measured the extent of conflict resolution at three points: (1) immediately after setting behaviors to an arbitrary initial configuration, (2) after we allowed the behavior of the agents to converge on a solution from an arbitrary initial configuration given only the original topological constraints, and (3) after we allowed the behavior of the agents to converge on a solution from an arbitrary initial configuration, but this time based on the modified topological constraints after a period of ritual-based self-optimization had taken place. Nevertheless, we still use the original topological constraints to measure conflict resolution even at this final point to make sure that the configurations found by the network are solutions to the original coordination problem.

We repeat these measures for 200 different arbitrary initial configurations at each point and calculated the average percentages of conflict resolution. We define the extent of conflict resolution as the percentage of all connections in which the behavioral states of the two connected agents are in agreement with each other. Each convergence from an arbitrary initial behavioral configuration to a solution consisted of 700 behavioral updates in total. For each update one arbitrary agent in the network is chosen. Overall each agent was allowed to update its behavioral state around 10 times on average (given that there are a total of 70 agents; 66 neighborhood leaders plus an additional 4 district leaders). This was generally sufficient for the network to reach a stable behavioral configuration.

The period of ritual-based self-optimization that occurs between points (2) and (3) of a run consisted of a series of 200 convergences, in which agents were allowed to adjust their connections by small amounts and to carry over these structural changes from one convergence to the next. For simplicity we implemented learning such that all agents synchronously update their connections at the end of a convergence. Similar to an accumulation of personal biases or habits, these structural adjustments change how the agents perceive the constraints of the original network topology, which means that the agents will start to behave differently with respect to each other.

4. Results

In order to enable a direct comparison between the original symmetrical scenarios and the extended, hierarchical scenarios, we calculated the extent of conflict resolution only in terms of the connections of the original “Districts” topology, while ignoring the connections introduced along with the four co-rulers.

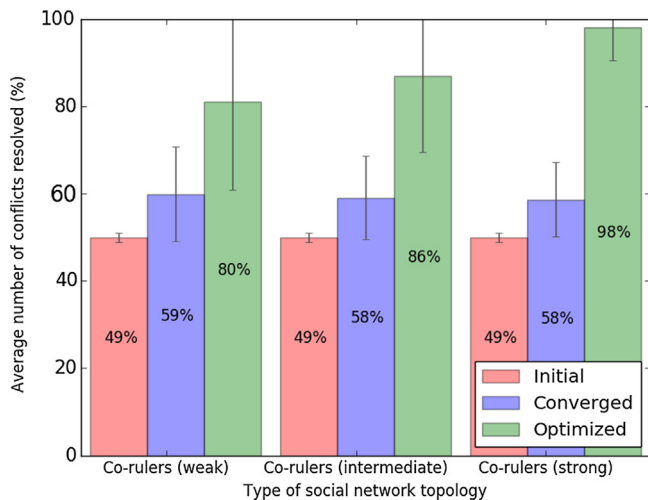


Fig. 4. Conflict resolution capacities of sociopolitical network with centralized hierarchy. A relation between two agents is defined to be in conflict when their behavior is not in agreement. The introduction of four co-rulers mitigates the conflicts caused by strong separation between quadrants, but only when two conditions hold: (1) they have an asymmetrically elevated influence over representatives of their district (intermediate and strong), and (2) there are community rituals (optimized). Otherwise their presence has no significant effect.

As can be appreciated in Fig. 4, the introduction of the co-rulers did not in itself have any effect on the network's capacity to simply converge on more cooperative configurations, no matter the relative strength of top-down influence (the converged configuration still only resolves 58–59% of conflicts). This null result was surprising because the inter-district linkage provided by the interaction among the co-rulers, especially combined with their strongly asymmetrical top-down influence, was expected to help coordinate the behaviors of the neighborhood leaders across the four districts. However, the presence of powerful co-rulers by themselves was not sufficient to improve the chances that leaders' behaviors converged on a network-wide consensus.

Instead there was a notable synergetic effect when the social hierarchy introduced via the co-rulers was combined with the process of ritual-based self-optimization ("Optimized"). In that case we see a positive correlation between strength of asymmetrical top-down control and average percentage of conflicts resolved (no asymmetry, 80% - which was the same as the original "Districts" scenario optimized without co-rulers; more influential by a factor of five, 86%; more influential by a factor of ten, 98%).

This intriguing finding suggests that the role of the co-rulers at Teotihuacan perhaps was not so much to use their political power to directly coerce the behavior of those lower in the social hierarchy into consensus configurations, which in any case would have been a difficult and costly strategy to maintain in reality (Carballo et al., 2014, p. 103). Instead the role of co-rulers could have been more indirect: they may have provided a more effective means of unleashing the positive social transformations already

inherent in the practice of communal ritual, similar to the coordinating role played by managerial leaders in complex stateless societies which maintain social order via ritualization of behaviors (Stanish, 2017).

5. Discussion

These modeling results fit nicely with the tendency in central Mexico for urbanization and religion to be highly intertwined (Carballo, 2016; Manzanilla, 1992). Moreover, they help us to better appreciate the tangible benefits of an elite that performed a more ceremonial role rather than holding absolute political power. Although this model has been designed with the sociopolitical network of Teotihuacan in mind, its findings regarding the conditions of social learning may therefore help to inform our understanding of the process of urbanization in this region more generally. We expect the process to begin with an emphasis on community ritual with little evidence for centralized rulership. However, as settlement sizes began to increase and internal divisions became unavoidable, it is likely that social hierarchy also became more relevant. Elites will then have started taking on a central role in the realization and coordination of public rituals. We can see this role reflected in the architecture of Teotihuacan (Murakami, 2014). For example, leaders could have coordinated rituals taking place in the plazas from atop the adjoining temple platforms.

Of course, the extent of public participation in rituals taking place in the ceremonial core of Teotihuacan is debatable. The city's core is certainly characterized by exceptionally large public spaces, and our model provides one possible explanation of their function. Yet most rituals in the ceremonial core probably were more exclusive and would not have involved face-to-face interaction among the majority of the city's population. Moreover, many rituals were performed more locally at the compound and neighborhood level, as was shown for the case of Teopancazco (Manzanilla, 2009a, 2012, 2017a). However, our model does not require spatial contiguity of the participants. We only assumed that all participants interrupted their behavior in a temporally synchronized manner, which means that community involvement in a ritual could be extended to households across the entire city as long as their rituals were held at the same time as the presumably more restricted events taking place in the ceremonial core. Given that many household ritual implements are inflected by state ideology (Filini, 2015; Manzanilla, 1996), this integration of public and private rituals seems to be a plausible assumption. Something like this seems to be envisioned by Sanders and Evans (2006) for the final phase of Teotihuacan:

private courtyard rituals may have been coordinated, in apartment compounds all over the city. Given the height and centrality of the Pyramid of the Sun, the view from its summit would have allowed its priests to monitor such

activities as they took place in all the city's residential compounds, including that of the Street of the Dead Complex. One can imagine conch-shell trumpets sounding through the air or the heavy rumble of drums signaling the hours when residents of the compounds would turn to their altars to honor lineage progenitors, and the wisps of incense smoke rising from thousands of courtyards would bear their homage to the sky and serve as public testimony to their piety.” (p. 269)

6. Conclusions

The model supports the possibility that Teotihuacan initially had a highly distributed sociopolitical network that became more hierarchical as divisions within the city increased, even while mostly retaining its heterarchical organization. The modeling results suggest that increasing political centralization and social hierarchy could have mitigated the negative effects of increasing social divisions, albeit only effectively so alongside a continuation of collective practices of ritual integration. This proposal is consistent with how Teotihuacan often portrayed its leaders within the city, namely in terms of the religious functions they realized in service of the community rather than as specific individuals.

Perhaps the most intriguing hypothesis that we can derive from this new modeling work is that if Teotihuacan was indeed governed by some form of collective rule, this may not have necessitated the kind of powerful bureaucratic institutions that were characteristically employed by collective pre-modern states to manage problems of collective action (Blanton & Fargher, 2009). At least according to our model Teotihuacan could have done without these institutions as long as it was internally not too subdivided and had a finely calibrated system of transformative communal rituals that promoted social learning at the level of the sociopolitical system as a whole. In other words, while most of the current literature has assumed that Teotihuacan was a pre-modern state, and has therefore focused on debating whether its state institutions had supported individualized or collective rule, it may be more productive to ask at which point in its development we can say with certainty that we are dealing with an integrated state at all.

A lot of social complexity that is cited in favor of Teotihuacan statehood, such as large-scale coordinated planning, monumental architecture and long-distance trading/raiding, can already be achieved by stateless societies in which there are prominent leaders, who nevertheless ultimately have a managerial role without any substantial power over others' behavior (Stanish, 2017). On the other hand, even though we must be careful not to confuse absence of evidence with evidence of absence, it is interesting that at Teotihuacan there is a notable lack of compelling evidence for typical state-level institutions, such as a public records, centralized markets, money, and policing.

Could early Teotihuacan have been a scaled-up version of a complex stateless society, as suggested by the modeling results? This novel version of the collective rule hypothesis not only explains why the evidence for powerful rulers and other state-level institutions is so ambiguous, it also helps us to make better sense of the sociopolitical function of the many open plazas with their adjoining religious structures and their shared alignment with the rhythms of the stars.

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Declarations of interest

None.

Appendix A.

In this Appendix A we describe the equations underlying our model. We follow Watson et al. (2011) in using a type of network architecture first proposed by Hopfield (1982) on which to run a self-optimization algorithm that has been shown to be applicable to social systems (Davies, Watson, Mills, Buckley, & Noble, 2011; Froese et al., 2018). Each agent of the network can adopt one of two discrete behavioral states, $s_i = +1$ or -1 , which stands for a binary choice of action (do-a/do-b). All agents have their behavioral state initialized to one or the other state randomly with equal probability. We use an asynchronous updating rule, which means that at each step an agent is randomly selected from the network to update its behavioral state. The selected agent will choose the behavioral state that maximizes its own utility, u_i , which is defined as the weighted sum of its social interactions:

$$u_i = \sum_j^N \omega_{ij} s_i s_j$$

The connection weight ω_{ij} represents the importance for agent i of satisfying the particular constraint posed by its connection to agent j . The multiplication of behavioral states with the connection's weight means that if $\omega_{ij} > 0$ then agent i will benefit from imitating the behavior of

Table 1

Connection weights for the different types of network topology. Note that the first four columns of weights specify the symmetrical connections of an undirected graph ($\omega_{ij} = \omega_{ji}$). Agents' behavior in an undirected graph can be more or less constrained by each other depending on the magnitude of the weight, but only symmetrically so. The last two columns specify the weights of the asymmetrical connections of a directed graph ($\omega_{ij} \neq \omega_{ji}$). The asymmetry allows for an element of social hierarchy to be included, namely in the specific sense that some elite agents put more constraints on the behavior of other agents than those agents put on the elite in return.

	Intra three-temple complex	Inter three-temple complexes	Inter districts	Intra co-rulers	From district to co-ruler	From co-ruler to district
Neighborhoods	1	0.01	0.01	N/A	N/A	N/A
Districts	1	0.03	0.002	N/A	N/A	N/A
Co-rulers (weak)	1	0.03	0.002	1	0.03	0.03
Co-rulers (intermediate)	1	0.03	0.002	1	0.03	0.15
Co-rulers (strong)	1	0.03	0.002	1	0.03	0.3

agent j (such that both do-a or both do-b), while a negative ω_{ij} means that agent i will benefit from complementing the behavior of agent j (such that they either do-a and do-b, or do-b and do-a). We only used positive weights because this ensures that two network-wide solutions exist in principle, namely in the form of a complete consensus of doing a or b.

We distinguish two aspects of network connectivity. On the one hand, we specify and retain the weights of the initial network configuration, namely the original network topology, ω_{ij}^O , which consists of different configurations of positive weights. The precise magnitude of the weights is chosen so that the connections are representative of a sociopolitical scenario of interest. The parameters for these scenarios are summarized in Table 1.⁴

We also keep track of the changes that accumulate as agents selfishly update the weights of their interactions according to their learnt biases or habits, ω_{ij}^L . These weight changes are equivalent to simplified Hebbian learning in Hopfield neural networks (Watson, Mills, et al., 2011). While the original weights ω^O are static throughout a run, the learned weights ω^L depend on how the agents chose to modify their connections. For the purpose of determining the behavior of an agent at time step t , the sum of both the original and learnt weights make up the current weights of the network:

$$\omega_{ij}(t) = \omega_{ij}^O + \omega_{ij}^L$$

By separating the connection weights into these two components we can easily update an agent's state s_i depending on the combined, modified weights, while it is also possible to determine how the set of all behavioral states satisfies the constraints of the original weight space ω^O alone. It is this unmodified topology that we used to produce the percentages of conflict resolution presented in the Results section.

We assume that agents are selfish and rational albeit only with local knowledge and with a biased or habituated perception, which means that for each of their social con-

nections they assess whether increasing or decreasing its strength will increase their perceived individual utility. In other words, the consequences for their utility u_i implied by both $\Delta\omega_{ij}(t) = +r$ and $\Delta\omega_{ij}(t) = -r$ are considered, and whichever will increase individual utility the most is accepted. We fixed the learning rate r to be the same for all of the experiments ($r = 0.0015$). If neither change provides an increase the connection remains unchanged. For convenience a change is only applied once at the end of a convergence. Similar results would be obtained if a smaller learning rate were applied continuously as long as the system spends most of its time in a converged state (Watson, Mills, et al., 2011). Accordingly, if $s_i s_j > 0$ then $\omega_{ij}(t+1) = \omega_{ij} + r$. Alternatively, if $s_i s_j < 0$ then $\omega_{ij}(t+1) = \omega_{ij} - r$. Otherwise the weight remains unchanged.

It may be questioned whether real agents can behave as rationally as this, but the assumption of perfect rationality has little impact on the overall dynamics of the social network. Similar effects can be obtained by assuming that agents always behave in a habitual manner, such that the propensity of agent i to imitate (or instead to complement) agent j 's behavior will always be enhanced if agent i is currently imitating (or complementing) agent j 's behavior (Davies et al., 2011).

In contrast to the original model by Froese et al. (2018) we did not impose limits on the size of the learnt weights in order not to arbitrarily restrict the scope of the structural changes enacted by the agents. We also did not impose symmetrical connections, given the top-down constraints imposed by the co-rulers. Removing this latter restriction opens up the possibility that the system will exhibit other kinds of dynamics than just the convergence toward fixed-point equilibria that were preferred by Hopfield (1982). Further work is required to understand in more detail the mathematical basis of these new results, which go beyond the original attractor-based definition of the self-optimization algorithm by Watson et al. (2011). Preliminary investigations of this larger space of possible networks suggests that the principles of self-optimization can be generalized (see, e.g. Zarco & Froese, 2018), as is also confirmed by the current results. These advances are promising because they demonstrate that the self-optimization mechanism is not bound to the rather

⁴ During replication of the original study we noticed that Froese et al. (2018) made a mistake in reporting the parameters of their "District" scenario. The correct parameters are reported here.

restrictive formalism of the Hopfield network in which they were originally developed.

Appendix A. Supplementary material

Supplementary data to this article can be found online at <https://doi.org/10.1016/j.cogsys.2018.09.018>.

References

- Altschul, J. H. (1987). Social districts in Teotihuacan. In E. McClung de Tapia & E. C. Rattray (Eds.), *Teotihuacan: Nuevos datos, nuevas síntesis, nuevos problemas* (pp. 191–218). D.F., Mexico: Universidad Nacional Autónoma de México.
- Angulo, J. (1995). Teotihuacán: Aspectos de la cultura a través de su expresión pictórica. In B. De La Fuente (Ed.), *La Pintura Mural Prehispánica en México I: Teotihuacán. Tomo II: Estudios* (pp. 65–186). Mexico City: Instituto de Investigaciones Estéticas, UNAM.
- Angulo, J. (2007). Early Teotihuacan and its government. In V. L. Scarborough & J. E. Clark (Eds.), *The political economy of ancient mesoamerica: Transformations during the formative and classic periods* (pp. 83–100). Albuquerque, NM: University of New Mexico Press.
- Axelrod, R. (1984). *The evolution of cooperation*. New York: Basic Books.
- Blanton, R. E., & Fargher, L. F. (2009). Collective action in the evolution of pre-modern states. *Social Evolution & History*, 8(2), 133–166.
- Blanton, R. E., Feinman, G. M., Kowalewski, S. A., & Peregrine, P. N. (1996). A dual-processual theory for the evolution of Mesoamerican civilization. *Current Anthropology*, 37(1), 1–14.
- Cabrera Castro, R., Gómez-Chávez, S., & Gazzola, J. (2007). New findings of mural painting. In M. T. Uriarte & T. Falcón (Eds.), *Museo de Murales Teotihuacanos Beatriz De La Fuente* (pp. 229–239). Ciudad Universitaria, Mexico: Instituto de Investigaciones Estéticas.
- Carballo, D. M. (2016). *Urbanization and religion in ancient central Mexico*. New York, NY: Oxford University Press.
- Carballo, D. M., & Feinman, G. M. (2016). Cooperation, collective action, and the archeology of large-scale societies. *Evolutionary Anthropology*, 25, 288–296.
- Carballo, D. M., Roscoe, P., & Feinman, G. M. (2014). Cooperation and collective action in the cultural evolution of complex societies. *Journal of Archaeological Method and Theory*, 21, 98–133.
- Coe, M. D., & Koontz, R. (2013). *Mexico: From the olmecs to the aztecs*. London, UK: Thames & Hudson.
- Cowgill, G. L. (2008). Teotihuacan as an urban place. In A. G. Mastache, R. H. Cobean, Á. García Cook, & K. Hirth (Eds.), *El Urbanismo en Mesoamerica / Urbanism in Mesoamerica. Volume 2* (pp. 85–112). Mexico City, DF and University Park, PA: INAH and Pennsylvania State University.
- Cowgill, G. L. (2015). *Ancient teotihuacan: Early urbanism in Central Mexico*. New York, NY: Cambridge University Press.
- Crumley, C. L. (2003). Alternative forms of social order: Heterarchy, political economy, and the ancient Maya. In L. Vernon, F. Scarborough, & N. Dunning (Eds.), *Heterarchy, political economy, and the ancient Maya: The three rivers region of the East-Central Yucatán Peninsula* (pp. 136–145). Tucson, AZ: The University of Arizona Press.
- Davies, A. P., Watson, R. A., Mills, R., Buckley, C. L., & Noble, J. (2011). “If you can’t be with the one you love, love the one you’re with”: How individual habituation of agent interactions improves global utility. *Artificial Life*, 17(3), 167–182.
- Evans, S. T. (2006). Antecedents of the Aztec palace: Palaces and political power in Classic and Postclassic Mexico. In J. J. Christie & P. J. Sarro (Eds.), *Palaces and power in the Americas: From Peru to the Northwest Coast* (pp. 285–310). Austin, TX: University of Texas Press.
- Evans, S. T. (2016). Location and orientation of Teotihuacan, Mexico: Water worship and processional space. *Penn State University Occasional Papers in Anthropology*, 33, 52–121.
- Fargher, L. F., Heredia Espinoza, V. Y., & Blanton, R. E. (2011). Alternative pathways to power in late Postclassic Highland Mesoamerica. *Journal of Anthropological Archaeology*, 30, 306–326.
- Filini, A. (2015). Teotihuacan: Ritual economy, exchange and urbanization processes in Classic period Mesoamerica. *Economic Anthropology*, 2, 97–119.
- Froese, T. (2015). The ritualised mind alteration hypothesis of the origins and evolution of the symbolic human mind. *Rock Art Research*, 32(1), 90–97.
- Froese, T. (in press). Ritual anti-structure as an alternate pathway to social complexity? The case of ancient Teotihuacan, Central Mexico. *Material Religion*.
- Froese, T., Gershenson, C., & Manzanilla, L. R. (2018). Can government be self-organized? A mathematical model of the collective social organization of ancient Teotihuacan, Central Mexico. *Plos One*, 9(10), e109966. <https://doi.org/10.1080/17432200.2018.1485346>.
- Gómez Chávez, S. (2017). The underworld at Teotihuacan: The sacred cave under the Feathered Serpent Pyramid. In M. H. Robb (Ed.), *Teotihuacan: City of Water, City of Fire* (pp. 48–55): Fine Arts Museums of San Francisco de Young and University of California Press.
- Gómez-Chávez, S. (2012). Structure and organization of neighborhoods in the ancient city of Teotihuacan. In M. C. Arnauld, L. R. Manzanilla, & M. E. Smith (Eds.), *The neighborhood as a social and spatial unit in mesoamerican cities* (pp. 74–101). Tucson, AZ: The University of Arizona Press.
- Headrick, A. (2007). *The teotihuacan trinity: The sociopolitical structure of an ancient mesoamerican city*. Austin, TX: University of Texas Press.
- Hirth, K. (2008). Incidental urbanism: The structure of the Prehispanic city in Central Mexico. In J. Marcus & J. A. Sabloff (Eds.), *The ancient city: New perspectives on urbanism in the old and new world* (pp. 273–297). Santa Fe, NM: School for Advanced Research Press.
- Hopfield, J. J. (1982). Neural networks and physical systems with emergent collective computational abilities. *Proceedings of the National Academy of Sciences of the USA*, 79, 2554–2558.
- Kohler, T. A., Smith, M. E., Bogaard, A., Feinman, G. M., Peterson, C. E., Betzenhauser, A., ... Bowles, S. (2017). Greater post-Neolithic wealth disparities in Eurasia than in North America and Mesoamerica. *Nature*, 551, 619–622.
- Lansing, J. S. (2002). “Artificial societies” and the social sciences. *Artificial Life*, 8, 279–292.
- Manzanilla, L. R. (1992). The economic organization of the Teotihuacan priesthood: Hypotheses and considerations. In J. C. Berlo (Ed.), *Art, ideology, and the city of teotihuacan: A symposium at Dumbarton Oaks, 8th and 9th October 1988* (pp. 321–338). Washington, DC: Dumbarton Oaks.
- Manzanilla, L. R. (1996). Corporate groups and domestic activities at Teotihuacan. *Latin American Antiquity*, 7(3), 228–246.
- Manzanilla, L. R. (1997). Teotihuacan: Urban archetype, cosmic model. In L. R. Manzanilla (Ed.), *Emergence and change in early urban societies* (pp. 109–132). New York, NY: Plenum.
- Manzanilla, L. R. (2001). Gobierno corporativo en Teotihuacan: Una revisión del concepto ‘palacio’ aplicado a la gran urbe prehispánica. *Anales de Antropología*, 35, 157–190.
- Manzanilla, L. R. (2007). State formation in the New World. In G. M. Feinman & T. D. Price (Eds.), *Archaeology at the Millennium: A Sourcebook* (pp. 381–413). New York, NY: Springer, US.
- Manzanilla, L. R. (2008). La iconografía del poder en Teotihuacan. In G. Olivier (Ed.), *Símbolos de Poder en Mesoamérica* (pp. 111–131). Mexico City: Instituto de Investigaciones Históricas and Instituto de Investigaciones Antropológicas, Universidad Nacional Autónoma de México.
- Manzanilla, L. R. (2009a). Corporate life in apartment and *barrio* compounds at Teotihuacan, Central Mexico: Craft specialization, hierarchy, and ethnicity. In L. R. Manzanilla & C. Chapdelaine (Eds.), *Domestic life in prehispanic capitals: a study of specialization, hierarchy, and ethnicity* (pp. 21–42). Ann Arbor, MI: The University of Michigan Museum of Anthropology.

- Manzanilla, L. R. (2009b). Les palais et résidences de Teotihuacan. *Dossiers d'Archéologie*, H.-S. n° 17, 20–23.
- Manzanilla, L. R. (2012). Introducción. Teopancazco, un centro de barrio multiétnico de Teotihuacan, In L. R. Manzanilla (Ed.), *Estudios arqueométricos del centro de barrio de Teopancazco en Teotihuacan* (pp. 17–66). Mexico City: Universidad Nacional Autónoma de México.
- Manzanilla, L. R. (2015). Cooperation and tensions in multiethnic corporate societies using Teotihuacan, Central Mexico, as a case study. *Proceedings of the National Academy of Sciences of the USA*, 112(30), 9210–9215.
- Manzanilla, L. R. (2017a). Teopancazco: A multiethnic neighborhood center in the metropolis of Teotihuacan. In L. R. Manzanilla (Ed.), *Teopancazco: investigations of a teotihuacan neighborhood center* (pp. 1–48). Gainesville, FL: University Press of Florida.
- Manzanilla, L. R. (2017b). *Teotihuacan, Ciudad Excepcional de Mesoamérica*. Mexico City: El Colegio Nacional.
- Manzanilla, L. R. (2017c). The Xalla palace in Teotihuacan. In M. H. Robb (Ed.), *Teotihuacan: City of Water, City of Fire* (pp. 118–123). San Francisco, CA: Fine Arts Museums of San Francisco de Young and University of California Press.
- Manzanilla, L. R., & López Luján, L. (2001). Exploraciones en un posible palacio de Teotihuacan: El Proyecto Xalla (2000–2001). *Mexicon*, XIII (3), 58–61.
- Mezza-Garcia, N., Froese, T., & Fernández, N. (2014). Reflections on the complexity of ancient social heterarchies: Toward new models of social self-organization in pre-Hispanic Colombia. *Journal of Sociocybernetics*, 12, 3–17.
- Millon, R. (1970). Teotihuacán: Completion of map of giant ancient city in the Valley of Mexico. *Science*, 170, 1077–1082.
- Millon, R. (1973). *Urbanization at Teotihuacan, Mexico. The Teotihuacan Map. Part One: Text*. Austin, TX: University of Texas Press.
- Millon, R. (1988). Where do they all come from? The provenance of the Wagner murals from Teotihuacan. In K. Berrin (Ed.), *Feathered serpents and flowering trees: reconstructing the murals of teotihuacan* (pp. 78–113). Seattle, WA: The Fine Arts Museums of San Francisco.
- Millon, R. (1993). The place where time began: An archaeologist's interpretation of what happened in Teotihuacan history. In K. Berrin & E. Pasztory (Eds.), *teotihuacan: art from the city of the gods* (pp. 16–43). London, UK: Thames & Hudson.
- Murakami, T. (2014). Social identities, power relations, and urban transformations: Politics of plaza construction at Teotihuacan. In K. Tsukamoto & T. Inomata (Eds.), *Mesoamerican plazas: arenas of community and power* (pp. 34–49). Tucson, AZ: University of Arizona Press.
- Nichols, D. L. (2016). Teotihuacan. *Journal of Archaeological Research*, 24, 1–74.
- Nielsen, J. (2014). Where kings once ruled? Considerations on palaces and rulership at Teotihuacan. In J. Nehammer Knub, C. Helmke, & J. Nielsen (Eds.), *Palaces and courtly culture in ancient mesoamerica. Archaeopress* (pp. 1–16). Oxford, UK: Archaeopress.
- Nielsen, J., & Helmke, C. (2017). Los bebedores de Tetitla. Representaciones del consumo ritual de pulque en los murales de Teotihuacan. In L. Staines Cicero & C. Helmke (Eds.), *Las Pinturas Realistas de Tetitla, Teotihuacan. Estudios a través de la obra de Agustín Villagra Caletí* (pp. 135–163). Mexico City: Instituto de Investigaciones Estéticas, UNAM and INAH.
- Ortman, S. G., Cabaniss, A. H. F., Sturm, J. O., & Bettencourt, L. M. A. (2014). The pre-history of urban scaling. *Plos One*, 9(2), e87902. <https://doi.org/10.1371/journal.pone.0087902>.
- Pasztory, E. (1988). A reinterpretation of Teotihuacan and its mural painting tradition. In K. Berrin (Ed.), *Feathered serpents and flowering trees: reconstructing the murals of teotihuacan* (pp. 45–77). Seattle, WA: The Fine Arts Museums of San Francisco.
- Pasztory, E. (1997). *Teotihuacan: An experiment in living*. Norman, OK: University of Oklahoma Press.
- Paulinyi, Z. (1981). Capitals in pre-Aztec central Mexico. *Acta Orientalia Academiae Scientiarum Hungaricae*, 35(2–3), 315–350.
- Paulinyi, Z. (2001). Los señores con tocado de borlas: Un estudio sobre el estado teotihuacano. *Ancient Mesoamerica*, 12, 1–30.
- Paulinyi, Z. (2014). The butterfly bird god and his myth at Teotihuacan. *Ancient Mesoamerica*, 25(1), 29–48.
- Plunket, P., & Uruñuela, G. (1998). Preclassic household patterns preserved under volcanic ash at Tetimpa, Puebla, Mexico. *Latin American Antiquity*, 9, 287–309.
- Robb, M. H. (Ed.). (2017). *Teotihuacan: City of Water, City of Fire*. San Francisco, CA: Fine Arts Museums of San Francisco de Young and University of California Press.
- Robertson, I. G. (2015). Investigating Teotihuacan through TMP surface collections and observations. *Ancient Mesoamerica*, 26, 163–181.
- Sanders, W. T., & Evans, S. T. (2006). Rulership and palaces at Teotihuacan. In J. J. Christie & P. J. Sarro (Eds.), *Palaces and Power in the Americas: From Peru to the Northwest Coast* (pp. 256–284). Austin, TX: University of Texas Press.
- Smith, M. E. (2017). The Teotihuacan anomaly: The historical trajectory of urban design in ancient Central Mexico. *Open Archaeology*, 3, 175–193.
- Stanish, C. (2017). *The evolution of human co-operation: Ritual and social complexity in stateless societies*. Cambridge, UK: Cambridge University Press.
- Sugiyama, N., Sugiyama, S., & Sarabia, G. A. (2013). Inside the sun pyramid at teotihuacan, Mexico: 2008–2011 excavations and preliminary results. *Latin American Antiquity*, 24(4), 403–432.
- Sugiyama, S. (2005). *Human sacrifice, militarism, and rulership: materialization of state ideology at the Feathered Serpent Pyramid, Teotihuacan*. Cambridge, UK: Cambridge University Press.
- Turner, V. (1969). *The ritual process: structure and anti-structure*. Piscataway, NJ: AldineTransaction.
- Ulloa, R., & Froese, T. (2016). Nobility-targeting raids among the Classic Maya: Cooperation in scale-free networks persists under tournament attack when population size fluctuates. In C. Gershenson, T. Froese, J. M. Siqueiros, W. Aguilar, E. J. Izquierdo, & H. Sayama (Eds.), *Proceedings of the artificial life conference* (pp. 472–479). Cambridge, MA: MIT Press.
- van Gennep, A. (1908/1960). *The rites of passage*. Chicago, IL: The University of Chicago Press.
- Villa Córdova, T. (2016). *La Vasija de Las Colinas: Significado, Poder y Culto a los Antepasados en Teotihuacán*. Mexico City, DF: INAH.
- Watson, R. A., Buckley, C. L., & Mills, R. (2011). Optimization in “self-modeling” complex adaptive systems. *Complexity*, 16(5), 17–26.
- Watson, R. A., Mills, R., & Buckley, C. L. (2011). Global adaptation in networks of selfish components: Emergent associative memory at the system scale. *Artificial Life*, 17(3), 147–166.
- Wenger, E. (2000). Communities of practice and social learning systems. *Organization*, 7(2), 225–246.
- Widmer, R. J., & Storey, R. (2012). The “Tlajinga Barrio”: A distinctive cluster of neighborhoods in Teotihuacan. In M. C. Arnauld, L. R. Manzanilla, & M. E. Smith (Eds.), *The neighborhood as a social and spatial unit in mesoamerican cities* (pp. 103–116). Tucson, AZ: The University of Arizona Press.
- Zarco, M., & Froese, T. (2018). Self-modeling in Hopfield neural networks with continuous activation function. *Procedia Computer Science*, 123, 573–578.